Final Examination
Business Mathematics

20^{th} February 2006

Time: 3 hours (9–12am)

1.

a. Give the definition of a homogeneous function. [2]

b. What are *order* and *degree* of a differential equation? [2] Given the differential equation

$$y^{(iv)} + 4\left(y''\right)^2 + \left(y'\right)^3 = \sin x$$

Give its degree and order. [2] Is this equation homogeneous? [1]

c. A *primitive* gives rise to a *differential equation*. Explain how the primitive $y = Ax^2 + Bx + C$ gives rise to the differential equation $\frac{d^3x}{dx^3} = 0$. [3]

2.

a. Explain what *sequence* and *series* are. When are they said to be *arithmetic*?, when *geometric*? [6]

b. Given an arithmetic series, $1 + 3 + 5 + 7 + \cdots$. What is the value of the n^{th} term? Find the value of the 50^{th} term. What is the sum of the first n term? Find the sum of the first 50 terms. [4]

c. Given a geometric series, $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$. What is the value of the n^{th} term? Find the value of the 50^{th} term. What is the sum of the first n term? Find the sum of the first 50 terms. What is the sum to infinity, that is to say, S_∞? [5]

3.

a. Give the formula for finding the present value of compound interest. Let the annual interest rate be 10 per cent. What is the present value of 1,000 Bahts in ten years' time? [4]

b. What is the present value at the end of n conversion periods in t years? Here m is the number of conversion periods per year. Find the present value of 1,000 Bahts in five years' time, when $m = 5$ and $i = 10$ per cent. [4]

c. Compare the present values obtained from (a) and (b), and discuss the difference between them. [2]

4.

a. What is the *average* or *mean value* of an integrable function $f(x)$ on $[a, b]$? [2]

b. What is an integrable function? [2] Give an example of a function that is not integrable on some range. [2]

c. Consider a (2×2) system of linear equations in the slope-intercept form,

$$y = m_1 x + b_1$$
$$y = m_2 x + b_2$$

Give conditions for the system to have a unique solution, no solutions, and infinitely many solutions. [4]

5.

a. Explain *revenue* and *elasticity* mathematically and verbally. [1] Explain *marginal* and *average costs* both mathematically and also in words. [1]

b. Give the condition for $a^0 = 1$ to be true. [1]

c. The following is a formal definition of limit.

For a function $\mathrm{f}(x)$, $\lim_{n \to a} \mathrm{f}(x) = l$ if and only if for every $\epsilon > 0$, there exists $\delta > 0$ such that $|\mathrm{f}(x) - l| < \epsilon$ whenever $0 < |x - a| < \delta$.

Explain in words what we mean by this. Give your answer both in English and in Thai. [1]

d. What is a *cut algorithm*? [1] In the following solved example, identify all pivot elements and explain in detail how the problem was solved. [5]

$$\begin{aligned} \text{maximise:} \quad & z = 2x_1 + x_2 \\ \text{subject to:} \quad & 2x_1 + 5x_2 \leq 17 \\ & 3x_1 + 2x_2 \leq 10 \\ \text{with:} \quad & x_1, x_2 \text{ non-negative and integral} \end{aligned}$$

Use cut algorithm.

Solution

Find the first approximation of Programme 1 normally using the simplex method.

		x_1	x_2	x_3	x_4	
		2	1	0	0	
x_3	0	2	5	1	0	17
x_4	0	3	2	0	1	10
		-2	-1	0	0	0

Since $\frac{10}{3} < \frac{17}{2}$, we know that 3 is the pivot element, and therefore we replace the basic variable x_4 with x_1.

	x_1	x_2	x_3	x_4	
x_3	0	$\frac{11}{3}$	1	$-\frac{2}{3}$	$\frac{31}{3}$
x_1	1	$\frac{2}{3}$	0	$\frac{1}{3}$	$\frac{10}{3}$
	0	$\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{20}{3}$

We have $x_1^* = \frac{10}{3}$, $x_3^* = \frac{31}{3}$, $x_2^* = x_4^* = 0$ and $z^* = \frac{20}{3}$. Since both x_1^* and x_3^* are non-integers, arbitrarily choose the former to generate a new constraint. Then our Programme 2 becomes,

$$\begin{aligned} x_1 + \frac{2}{3}x_2 + \frac{1}{3}x_4 &= \frac{10}{3} = 3 + \frac{1}{3} \\ \frac{2}{3}x_2 + \frac{1}{3}x_4 - \frac{1}{3} &= 3 - x_1 \\ \frac{2}{3}x_2 + \frac{1}{3}x_4 - \frac{1}{3} &\geq 0 \\ \frac{2}{3}x_2 + \frac{1}{3}x_4 &\geq \frac{1}{3} \\ 2x_2 + x_4 &\geq 1 \end{aligned}$$

and our new programme becomes

$$\begin{aligned} \text{maximise:} \quad & z = 2x_1 + x_2 + 0x_3 + 0x_4 \\ \text{subject to:} \quad & \frac{11}{3}x_2 - \frac{2}{3}x_4 = \frac{31}{3} \\ & x_1 + \frac{2}{3}x_2 - \frac{1}{3}x_4 = \frac{10}{3} \\ & 2x_2 + x_4 \geq 1 \\ \text{with:} \quad & \text{all variables non-negative and integral} \end{aligned}$$

		x_1	x_2	x_3	x_4	x_5	x_6	
		2	1	0	0	0	$-M$	
x_1	0	1	$\frac{2}{3}$	0	$\frac{1}{3}$	0	0	$\frac{10}{3}$
x_3	0	0	$\frac{11}{3}$	1	$-\frac{2}{3}$	0	0	$\frac{31}{3}$
x_6	$-M$	0	2	0	1	-1	1	1
		-2	-1	0	0	0	0	0
		0	-2	0	-1	1	-1	-1

Now x_2 replaces x_6 in the basic variables and becomes the pivot element.

	x_1	x_2	x_3	x_4	x_5	x_6	
x_1	1	0	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	3
x_3	0	0	1	$-\frac{15}{6}$	$\frac{11}{6}$	$-\frac{11}{6}$	$\frac{17}{2}$
x_2	0	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	-2	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
	0	0	0	0	0	0	0

This becomes,

	x_1	x_2	x_3	x_4	x_5	
x_1	1	0	0	0	$\frac{1}{3}$	3
x_3	0	0	1	$-\frac{5}{2}$	$\frac{11}{6}$	$\frac{17}{2}$
x_2	0	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
	0	0	0	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{13}{2}$

Then our first approximation of Programme 2 is $x_1^* = 3$, $x_2^* = \frac{1}{2}$, $x_3^* = \frac{17}{2}$, $x_4^* = x_5^* = 0$, and $z^* = \frac{13}{2}$. Arbitrarily choose x_2^* to generate the new constraint.

$$\begin{aligned} x_2 + \frac{1}{2}x_4 - \frac{1}{2}x_5 &= \frac{1}{2} \\ \frac{1}{2}x_4 - \frac{1}{2}x_5 - \frac{1}{2} &= -x_2 \\ \frac{1}{2}x_4 - \frac{1}{2}x_5 - \frac{1}{2} &\geq 1 \\ x_4 - x_5 &\geq 1 \end{aligned}$$

Then our Programme 3 becomes,

$$\begin{aligned} \text{maximise:} \quad & z = 2x_1 + x_2 + 0x_3 + 0x_4 + 0x_5 \\ \text{subject to:} \quad & x_1 + \frac{1}{3}x_5 = 3 \\ & x_3 - \frac{5}{2}x_4 + \frac{11}{6}x_5 = \frac{17}{2} \\ & x_2 + \frac{1}{2}x_4 - \frac{1}{2}x_5 = \frac{1}{2} \\ & x_4 - x_5 \geq 1 \\ \text{with:} \quad & \text{all variables non-negative and integral} \end{aligned}$$

We draw our table for this programme.

		x_1	x_2	x_3	x_4	x_5	x_6	x_7	
		2	1	0	0	0	0	$-M$	
x_1	0	1	0	0	0	$\frac{1}{3}$	0	0	3
x_2	0	0	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$
x_3	0	1	0	0	$-\frac{5}{2}$	$\frac{11}{6}$	0	0	$\frac{17}{2}$
x_7	$-M$	0	0	0	1	-1	-1	1	1
		-2	-1	0	0	0	0	0	0
		0	0	0	-1	1	1	-1	-1

Then x_4 replaces the basic x_7 to become the pivot element.

	x_1	x_2	x_3	x_4	x_5	x_6	
x_1	1	0	0	0	$\frac{1}{3}$	0	3
x_2	0	1	0	0	0	$\frac{1}{2}$	0
x_3	1	0	0	0	$-\frac{2}{3}$	$-\frac{5}{2}$	11
x_4	0	0	0	1	-1	-1	1
	-2	-1	0	0	0	0	0

Next, x_1 remains basic and becomes a pivot element.

	x_1	x_2	x_3	x_4	x_5	x_6	
x_1	1	0	0	0	$\frac{1}{3}$	0	3
x_2	0	1	0	0	0	$\frac{1}{2}$	0
x_3	0	0	0	0	-1	$-\frac{5}{2}$	8
x_4	0	0	0	1	-1	-1	1
	0	-1	0	0	$\frac{1}{3}$	0	6

This becomes

	x_1	x_2	x_3	x_4	x_5	x_6	
x_1	1	0	0	0	$\frac{1}{3}$	0	3
x_2	0	1	0	0	0	$\frac{1}{2}$	0
x_3	0	0	0	0	-1	$-\frac{5}{2}$	8
x_4	0	0	0	1	-1	-1	1
	0	0	0	0	$\frac{1}{3}$	$\frac{1}{2}$	6

The optimum point for Programme 3 is then, $x_1^* = 3$, $x_3^* = 8$, $x_4^* = 1$, $x_2^* = x_5^* = x_6^* = 0$ and $z^* = 6$. Therefore the solution to the original problem Programme 1 is $x_1^* = 3$, $x_2^* = 0$ at the objective value $z^* = 6$.

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